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ABSTRACT

This paper links the conceptions of an experienced high school mathematics teacher to aspects of his first implementation of a reform-oriented curriculum during a 6-week functions unit. The teacher exhibited comprehensive understandings of the function concept, dominated by graphical representations and a covariation description of function. These features contributed to classroom emphases on the use of multiple representations to understand dependence patterns in data and characteristics of different types of relationships. (Author)

The Role of Teacher's Mathematical Conceptions in His Implementation of a Reform-Oriented Functions Unit

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THE ROLE OF ONE TEACHER'S MATHEMATICAL CONCEPTIONS IN HIS IMPLEMENTATION OF A REFORM-ORIENTED FUNCTIONS UNIT

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This paper links the conceptions of an experienced high school mathematics teacher to aspects of his first implementation of a reform-oriented curriculum during a six-week functions unit. The teacher exhibited comprehensive understandings of the function concept, dominated by graphical representations and a covariation description of function. These features contributed to classroom emphases on the use of multiple representations to understand dependence patterns in data and characteristics of different types of relationships.

The current reform movement in mathematics education places new demands on teachers to offer students varied classroom opportunities to develop deep understandings of the function concept. The envisioned treatment of functions in grades 9-12 includes modeling real-world situations using functions; representations and interpretations of relationships using tables, graphs, equations, and verbal descriptions; translations between multiple representations of functions; and recognition of the variety of problem situations that can be modeled by the same type of function (National Council of Teachers of Mathematics [NCTM], 1989). There is little empirical information about how secondary mathematics teachers cope with the complexity of change as they attempt to incorporate such recommendations into their teaching practice. This paper documents the influence of one veteran high school teacher's mathematical conceptions on his instruction during a six-week functions unit using the reform-oriented curricular materials of the Core-Plus Mathematics Project.

Previous empirical and theoretical work about teachers' and students' understandings of the function topic contributed to the conceptualization of this study (e.g. Even, 1990; Leinhardt, Zaslavsky, & Stein, 1990; Norman, 1992; Vinner & Dreyfus, 1989). Our framework also builds on the growing body of research and theory related to teachers' knowledge and beliefs about mathematics and teaching that has begun to support the notion that teachers' conceptions contribute significantly to their instructional practice (Fennema & Franke, 1992). In particular, recent investigations have reported that experienced teachers' knowledge acts as a critical filter in the interpretation of reform-oriented mathematics curricula (Gamoran, 1994; Wilson, 1990). Taken as a whole, this literature emphasizes the need for further consideration of the complex relationship between teachers' conceptions and their instruction in the mathematics classroom.

Research Design

This study followed an interpretive case study design to investigate the conceptions of Mr. Allen, a 14-year veteran high school mathematics teacher, as part of a larger, ongoing project examining the experiences of three teachers imple-

menting the Core-Plus materials for the first time. During the 1994-95 school year, Mr. Allen voluntarily used the Core-Plus Course 1 materials in a single class of ninth grade students. The public school district where he teaches is located in a small urban community in the Northeast United States. Data were collected between September 1994 and January 1995 using interviews, observations, and classroom artifacts. All interview data were transcribed.

At the beginning of the school year, Mr. Allen participated in two video-taped baseline interviews. In the first interview, referred to as the function sort, he was given the task of interpreting and organizing a stack of 32 cards depicting mathematical relationships that varied along these dimensions: family of function, representation, and particular characteristics such as functionality or continuity. These differences supplied a challenging set of situations for Mr. Allen to analyze and multiple criteria on which to base an arrangement of the cards. The second baseline interview was used to further investigate his informal and formal descriptions of the function concept and his orientations toward teaching about functions.

Mr. Allen's classroom was observed on a daily basis for 26 consecutive lessons (11/11/94 - 12/22/94) while he implemented the Core-Plus *Patterns of Change* unit that focuses on varied representations and explorations of real-world functional relationships. Detailed fieldnotes of his teaching were taken, and worksheets, quizzes, and tests were collected. A videocamera and remote microphone followed Mr. Allen as he moved around the classroom, capturing both whole-class and small group instruction. Observations were supplemented by four interviews during which Mr. Allen was asked to comment on certain instructional decisions. At the conclusion of his instruction with *Patterns of Change*, he watched selected videotaped segments from the function sort interview and reflected on his experience with the unit as it related to the particular segments.

Mr. Allen's Conceptions Prior to Teaching the Unit

Given a choice of formal textbook definitions of function including both correspondence and covariation descriptions, Mr. Allen favored definitions involving univalent correspondences between sets. However, his informal characterization of function involved "the relationship between two things and ... how a change in one affects the other," a covariation relationship that he claimed is most clearly viewed in a graphical display. Despite his contrasting concept image and definition, Mr. Allen demonstrated flexibility in his thinking as he appreciated the different utilities of the two notions of function: his covariation image describes "how the two variables are working together," but the more restrictive univalent correspondence definition is indispensable because "how [the variables] work together determines whether it's a function or not." Although Mr. Allen made occasional use of his formal concept definition, for example to determine the functionality of unfamiliar relationships, his covariation image dominated his verbalizations and actions in the function sort interview, during which he repeatedly asked himself the question, "As one changes, what happens to the other one?" and relied heavily on graphical representations to interpret the function sort situations. His analysis

of each relationship consisted of first developing a general sense of how a change in one variable affects the other, and then attaching a label that indicates the appropriate family of function. For instance, to understand a verbal description of the area and diameter of a pizza, he first expressed that "as the diameter would increase, the area would increase by that factor to the second power," and then concluded that the relationship was "quadratic."

Mr. Allen's efforts to develop appropriate labels also illustrate his graphical proficiency. When he examined cards showing graphs, he created family labels (e.g. linear, quadratic, etc.) on sight by immediate recognition. In contrast, he found the information in verbal and tabular representations to be less accessible and thus made frequent translations to graphs before labeling them. For example, although examination of the table shown in Figure 1 led Mr. Allen to observe that there are "some square numbers on the bottom," he did not achieve a conclusive family label until he created a graph to help him "see" the relationship.

x	-2	-1	0	1	2
y	1	0	1	4	9

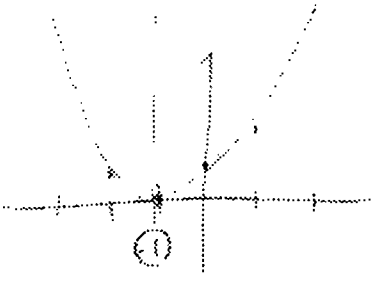

Figure 1. A table of "perfect squares" and Mr. Allen's corresponding graph.

Looking at his graph, he announced that "it's parabolic centered around negative 1," and concluded that the table represents a quadratic relationship. Mr. Allen's visual strengths also facilitated his construction of a card ordering, based on traditional teaching sequence, that began in the cards showing graphs and resulted in similar core orderings in each of the four representations of the sort as follows: linear, polynomial, exponential and logarithmic, and trigonometric. The consequent organization allowed Mr. Allen to point out connections between different representations of the same family of function.

To summarize briefly, covariation and graphical representations dominate Mr. Allen's thinking about functional situations. Covariation notions guide him toward the development of a complete description of a dependence pattern, including association with a major family of function. Graphs offer Mr. Allen the best display of covariation, serve as the primary format to which other representations are translated for analysis, and thus act as the source for connections between different representations of the same family of function.

Links Between Mr. Allen's Conceptions and Instruction

Mr. Allen's strong covariation image played a crucial supporting role in his adaptation to the Core-Plus approach to functions that is summarized in this ex-

cerpt from the *Patterns of Change* Student Text: "In many cases, we can describe the relation between two variables by saying that one variable is a function of another, particularly if the value of one variable depends on the other" (p. 4). Although his formal correspondence definition and typical classroom introduction of function appeared to be closely tied to traditional materials and activities, Mr. Allen's flexible understandings enabled him to comfortably enact the less formal covariation approach laid out in *Patterns of Change* and place emphasis on dependence relationships throughout his instruction. He repeatedly engaged students in discussions framed by the very same questions that he had applied to his own thinking during the function sort: "*Is* there a relationship?" and "*How* are the variables related?" The use of these questions put his own images into action in the creation of opportunities to encourage students to interpret features of a variety of dependence relationships.

Consistent with the dominance of visual representations in his own thinking about functions, Mr. Allen gave precedence to graphs in his implementation of the unit, portraying them as crucial tools that offer the optimal display of patterns of dependence. As he pointed out to a student, "The table gives you times and heights, but the graph gives you *the relationship* between time and height." The privileged position of graphs among the representations he used in the classroom was further evidenced in his numerous additions of "investigative graphing" tasks to assignments, and his urging of students to make effective use of the graphics calculators to quickly create visual representations.

Although Mr. Allen's conceptions and instruction were dominated by graphical displays of relationships, his classroom actions also demonstrated a high regard for explorations of multiple representations of problem situations. Mr. Allen's overriding interest in the determination of covariation patterns contributed to his tolerance of representations other than graphs. He appreciated the centrality of the variety of tables, graphs, equations, and verbal descriptions in the *Patterns of Change* activities because of the different information that each representation provides. As he explained, although graphs are the most helpful to him personally, "some people might be able to see the relationship with an equation, and ... making a table can maybe help them." In accord with his belief that further representations offer increased opportunities for students to understand a relationship, Mr. Allen frequently supplemented the *Patterns of Change* materials with extra representational tasks, including development of both recursive and explicit rules, construction of tables and graphs, and writing verbal descriptions of covariation patterns.

In addition to his classroom focus on the variety of perspectives provided by multiple representations, Mr. Allen emphasized the links between different displays of the same relationship. Reflecting the strength of the connections that he exhibited during the baseline interviews, Mr. Allen repeatedly communicated to students that the "rule, table, and graph all show the same thing." He capitalized on graphs in his instruction as a starting point from which to stress connections between multiple representations of the same situation, and to accentuate the features that distinguish different families of functions.

The main themes just discussed may be best illustrated by an example of Mr. Allen's classroom interactions that occurred during a *Patterns of Change* lesson involving the development of tables, graphs, and the explicit equations I=2.50T and P=2.50T-450 relating a theater's daily income I and profit P to the number of tickets sold T. After students had investigated this situation in their groups, Mr. Allen used the two linear graphs for I and P versus T (as shown in Figure 2) as a focal point for a whole-class summary discussion.

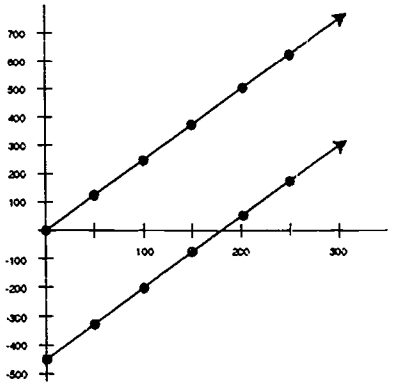

Figure 2. Reproduction of Mr. Allen's display of two graphs on the chalkboard.

Mr. Allen first drew attention to the constant rate of change in the covariation relationship between I and T by noting the increasing graph and asking, "*How* is it going up? If you sell 1 ticket, how much income do you get?" This ongoing form of questioning allowed him to highlight that "it is going up by the same amount each time," and to subsequently connect the constantly increasing data values to the linearity of the graph: "That's why you see the dots lining up." Having illustrated the constant rate of change in I, Mr. Allen attempted to demonstrate the same feature in P by asking "Does anyone notice anything about these two lines?" Mr. Allen related the students' visual observations that the lines are straight, parallel, and increasing back to the table data and equations: "For every ticket you sell, both are going up by 2.50 for each ticket sold. Notice in the equations, both are multiplied by 2.50." In a similar manner, he pointed out the different "starting points" for I and P through comparison of the graphs and equations. Thus with his demonstrations rooted in graphical displays, Mr. Allen identified distinguishing features of linearity and established links between their emergence in the varied representations of the theater situation.

The above illustration exemplifies the tight connections between Mr. Allen's conceptions and his instructional emphases that were evidenced throughout his implementation of the unit. His graphical proficiency and personal focus on patterns of covariation empowered him to utilize the Core-Plus materials to facilitate classroom interactions that constructed explicit ties between representations and types of functional relationships.

Discussion

Our findings corroborate those of other studies suggesting that teachers' comprehensive, well-organized conceptions contribute to instruction characterized by

an emphasis on conceptual connections, powerful representations, and meaningful discussions. In stark contrast to the teacher in the study of Stein, Baxter, and Leinhardt (1990) whose limited knowledge of functions led to narrow instruction marked by *missed opportunities* to highlight connections between concepts and representations, Mr. Allen applied his flexible understandings to his implementation of the Core-Plus unit in ways that *created* such opportunities. Moreover, this study illustrates the notion that teachers who can make connections between different approaches to content can adjust their teaching to accommodate ideas that are not traditionally emphasized in the school curriculum. Because Mr. Allen was able to reconcile the Core-Plus approach to functions with the prominent features of his own conceptions of functions, the *Patterns of Change* materials furnished a way for him to translate his understandings into new but comfortable classroom strategies.

This paper focuses on the impact of Mr. Allen's conceptions of function on his instruction with the Core-Plus materials, but there were certainly other important influences on his teaching. For instance, throughout the year, he faced a tension between teacher direction and student independence. As Mr. Allen himself suggested, finding a suitable balance will involve developing greater familiarity with the Core-Plus materials and classroom behavior in a more student-centered classroom. The resolution of this struggle is particularly important in light of Mr. Allen's deep conceptions: the more comfortable he can become with his newly-defined role as teacher in the reform classroom, the more freedom and energy he will be able to devote to the application of his comprehensive understandings to even more meaningful opportunities for student learning.

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